12A

1(a) $\int_1^3 2x+5 \, dx$

$= \left[x^2 + 5x \right]_1^3$

$= \left((3)^2 + 5(3) \right) - \left((1)^2 + 5(1) \right)$

$= (24) - (6)$

$= \underline{\underline{18}}$

1(e) $\int_{-2}^2 4x^3 + x^2 - 3x + 2 \, dx$

$= \left[x^4 + \frac{x^3}{3} - \frac{3x^2}{2} + 2x \right]_{-2}^2$

$= \left((2)^4 + \frac{(2)^3}{3} - \frac{3(2)^2}{2} + 2(2) \right) - \left((-2)^4 + \frac{(-2)^3}{3} - \frac{3(-2)^2}{2} + 2(-2) \right)$

$= \left(16 + \frac{8}{3} - 6 + 4 \right) - \left(16 - \frac{8}{3} - 6 - 4 \right)$

$= \left(16\frac{2}{3} \right) - \left(3\frac{1}{3} \right)$

$= \underline{\underline{13\frac{1}{3}}}$

12A

2(b) $x(2x-1)^2$

$= x(2x-1)(2x-1)$

$= x(4x^2-2x-2x+1)$

$= 4x^3-4x^2+x$

$\int_{-2}^{1} x(2x-1)^2\,dx$

$= \int_{-2}^{1} 4x^3-4x^2+x\,dx$

$= \left[x^4-\frac{4x^3}{3}+\frac{x^2}{2}\right]_{-2}^{1}$

$= \left((1)^4-\frac{4(1)^3}{3}+\frac{(1)^2}{2}\right)-\left((-2)^4-\frac{4(-2)^3}{3}+\frac{(-2)^2}{2}\right)$

$= \left(1-\frac{4}{3}+\frac{1}{2}\right)-\left(16--\frac{32}{3}+\frac{4}{2}\right)$

$= \left(\frac{6}{6}-\frac{8}{6}+\frac{3}{6}\right)-\left(16+10\tfrac{2}{3}+2\right)$

$= \frac{1}{6}-28\tfrac{2}{3}$

$= \frac{1}{6}-28\tfrac{4}{6}$

$= -28\tfrac{3}{6}$

$= \underline{-28\tfrac{1}{2}}$

12A

3(d) $\int_4^9 3\sqrt{x}\,dx$

$= \int_4^9 3x^{1/2}\,dx$

$= \left[\frac{3x^{3/2}}{3/2}\right]_4^9$

$= \left[\frac{2}{3} \times 3x^{3/2}\right]_4^9$

$= \left[2x^{3/2}\right]_4^9$

$= \left[2\sqrt{x^3}\right]_4^9$

$= \left(2\sqrt{9^3}\right) - \left(2\sqrt{4^3}\right)$

$= \left(2 \times 3^3\right) - \left(2 \times 2^3\right)$

$= 54 - 16$

$= \underline{\underline{38}}$

12A

3(h) $\int_1^9 10\sqrt{x^3}\,dx$

$= \int_1^9 10x^{3/2}\,dx$

$= \left[\dfrac{10x^{5/2}}{5/2}\right]_1^9$

$= \left[\dfrac{2}{5} \times 10x^{5/2}\right]_1^9$

$= \left[4x^{5/2}\right]_1^9$

$= \left[4\sqrt{x^5}\right]_1^9$

$= \left(4\sqrt{9^5}\right) - \left(4\sqrt{1^5}\right)$

$= 4 \times 3^5 - 4 \times 1^5$

$= 4 \times 243 - 4 \times 1$

$= 972 - 4$

$= \underline{\underline{968}}$

12A

4 d) $\int_{-3}^{-1} (2x+3)^5 \, dx$

$= \left[\frac{(2x+3)^6}{2 \times 6} \right]_{-3}^{-1}$

$= \left[\frac{(2x+3)^6}{12} \right]_{-3}^{-1}$

$= \left(\frac{(2(-1)+3)^6}{12} \right) - \left(\frac{(2(-3)+3)^6}{12} \right)$

$= \frac{(1)^6}{12} - \frac{(-3)^6}{12}$

$(-3 \times -3) \times (-3 \times -3) \times (-3 \times -3)$

$9 \times 9 \times 9$

$= 729$

$= \frac{1}{12} - \frac{729}{12}$

$= \frac{-728}{12}$

$= -60\frac{8}{12}$

$= -60\frac{2}{3}$

12A

5(a) $\displaystyle\int_1^3 \frac{x^3-2}{x^2}\,dx$

$$= \int_1^3 \frac{x^3}{x^2} - \frac{2}{x^2}\,dx$$

$$= \int_1^3 x - 2x^{-2}\,dx$$

$$= \left[\frac{x^2}{2} - \frac{2x^{-1}}{-1}\right]_1^3$$

$$= \left[\frac{x^2}{2} + 2x^{-1}\right]_1^3$$

$$= \left[\frac{x^2}{2} + \frac{2}{x}\right]_1^3$$

$$= \left(\frac{(3)^2}{2} + \frac{2}{3}\right) - \left(\frac{(1)^2}{2} + \frac{2}{1}\right)$$

$$= \left(\frac{9}{2} + \frac{2}{3}\right) - \left(\frac{1}{2} + 2\right)$$

$$= \left(\frac{27}{6} + \frac{4}{6}\right) - \left(\frac{5}{2}\right)$$

$$= \left(\frac{31}{6}\right) - \left(\frac{15}{6}\right)$$

$$= \frac{16}{6}$$

$$= \frac{8}{3}$$

12A

7. $\int_{-2}^{k} 4x - 3 \, dx = -15$

Evaluate $\int_{-2}^{k} 4x - 3 \, dx$

$$= \left[\frac{4x^2}{2} - 3x\right]_{-2}^{k}$$

$$= \left(\frac{4k^2}{2} - 3k\right) - \left(\frac{4(-2)^2}{2} - 3(-2)\right)$$

$$= (2k^2 - 3k) - (8 + 6)$$

$$= 2k^2 - 3k - 14 \quad (= -15)$$

$$2k^2 - 3k - 14 = -15$$

$$2k^2 - 3k + 1 = 0$$

$$(2k-1)(k-1) = 0$$

$2k - 1 = 0 \qquad k - 1 = 0$

$2k = 1 \qquad \underline{\underline{k = 1}}$

$\underline{\underline{k = \frac{1}{2}}}$

12A

(8) (a) $\int_{-2}^{p} 3x^2 - 4x \, dx$

$= \left[x^3 - 2x^2\right]_{-2}^{p}$

$= (p^3 - 2p^2) - ((-2)^3 - 2(-2)^2)$

$= (p^3 - 2p^2) \quad -(-16)$

$= p^3 - 2p^2 + 16 \; (= 48)$ (told in question)

$\Rightarrow \quad p^3 - 2p^2 - 32 = 0$

(b)

4	1	−2	0	−32
		4	8	32
	1	2	8	0

$r = 0 \quad \therefore x = 4$ is a root and $(x-4)$ is a factor of $f(x)$.

$f(x) = (x-4)(x^2 + 2x + 8)$

$x^2 + 2x + 8 = 0$

$b^2 - 4ac$

$= (2)^2 - 4(1)(8)$

$= -30 < 0 \quad \therefore$ no real roots

$\therefore p = 4$ is the only real root of $f(x) = 0$ and $p = 4$ is the only real value for which $\int_{-2}^{p} 3x^2 - 4x \, dx = 48$.

12B

1(d) $\int_0^{\pi/6} 3\cos 2x \, dx$

$= \left[3 \times \frac{1}{2} \sin 2x \right]_0^{\pi/6}$

$= \left[\frac{3}{2} \sin 2x \right]_0^{\pi/6}$

$= \left(\frac{3}{2} \sin\left(2 \frac{\pi}{6}\right) \right) - \left(\frac{3}{2} \sin(0) \right)$

$= \frac{3}{2} \sin\left(\frac{\pi}{3}\right) - \frac{3}{2} \cancel{\sin(0)}^{0}$

$= \frac{3}{2} \cdot \frac{\sqrt{3}}{2} - 0$

$= \frac{3\sqrt{3}}{4}$

$\frac{12B}{2(b)}$

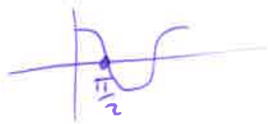
$$\int_0^{\pi/4} 6\sin\left(x+\frac{\pi}{4}\right)dx$$

$$= \left[6x - \cos\left(x+\frac{\pi}{4}\right)\right]_0^{\pi/4}$$

$$= \left[-6\cos\left(x+\frac{\pi}{4}\right)\right]_0^{\pi/4}$$

$$= \left(-6\cos\left(\frac{\pi}{4}+\frac{\pi}{4}\right)\right) - \left(-6\cos\left(0+\frac{\pi}{4}\right)\right)$$

$$= \left(-6\,\overset{0}{\cancel{\cos\left(\frac{\pi}{2}\right)}}\right) - \left(-6\cos\left(\frac{\pi}{4}\right)\right)$$



$$= (0) - \left(-6\cdot\frac{1}{\sqrt{2}}\right)$$

$$= \frac{6}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{6\sqrt{2}}{2}$$

$$= \underline{\underline{3\sqrt{2}}}$$

12B

3(c) $$\int_0^{\frac{5\pi}{4}} 2\cos\left(2x-\frac{\pi}{4}\right)dx$$

$$= \left[2 \times \frac{1}{2}\sin\left(2x-\frac{\pi}{4}\right)\right]_0^{\frac{5\pi}{4}}$$

$$= \left[\sin\left(2x-\frac{\pi}{4}\right)\right]_0^{\frac{5\pi}{4}}$$

$$= \left(\sin\left(2\left(\frac{5\pi}{4}\right)-\frac{\pi}{4}\right)\right) - \left(\sin\left(0-\frac{\pi}{4}\right)\right)$$

$$= \sin\left(\frac{9\pi}{4}\right) - \sin\left(-\frac{\pi}{4}\right)$$

$\sin\frac{9\pi}{4} = \sin\frac{\pi}{4}$

$\sin\left(-\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right)$

$$= \sin\frac{\pi}{4} - \left(-\sin\frac{\pi}{4}\right)$$

$$= \frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)$$

$$= \frac{2}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{2\sqrt{2}}{2}$$

$$= \underline{\underline{\sqrt{2}}}$$

12B

4(d) $\int_{-1.2}^{3.4} \frac{2}{3}\cos(2-x)\,dx$

$= \left[\frac{2}{3} \times \frac{1}{-1} \sin(2-x)\right]_{-1.2}^{3.4}$

$= \left[-\frac{2}{3}\sin(2-x)\right]_{-1.2}^{3.4}$

$= \left(-\frac{2}{3}\sin(2-3.4)\right) - \left(-\frac{2}{3}\sin(2--1.2)\right)$

$= \cancel{\frac{2}{3}}\ 0.62$

* remember - must be in radians

5) $\int_{\frac{\pi}{6}}^{t} 3\cos x\,dx$

$= \left[3\sin x\right]_{\pi/6}^{t}$

$= (3\sin t) - (3\sin \pi/6)$

$= 3\sin t - 3 \times \frac{1}{2}$

$= 3\sin t - \frac{3}{2}$

$\int_{\pi/6}^{t} 3\cos x\,dx = \frac{3\sqrt{2}}{2} - \frac{3}{2}$

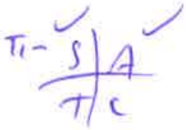
$\Rightarrow \quad 3\sin t - \frac{3}{2} = \frac{3\sqrt{2}}{2} - \frac{3}{2}$

$3\sin t = \frac{3\sqrt{2}}{2}$

12B

(5) continued

$$\sin t = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$



$$t = \frac{\pi}{4},\ \pi - \frac{\pi}{4}$$

$$t = \frac{\pi}{4},\ \frac{3\pi}{4}$$

(6) $\int_0^p \sin 2x \, dx$

$$= \left[-\frac{1}{2}\cos 2x\right]_0^p$$

$$= \left(-\frac{1}{2}\cos 2p\right) - \left(-\frac{1}{2}\overset{1}{\cancel{\cos 0}}\right)$$

$$= -\frac{1}{2}\cos 2p + \frac{1}{2}$$

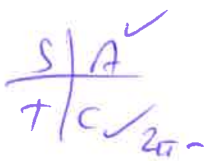
$$\int_0^p \sin 2x \, dx = \frac{1}{4}$$

$$\Rightarrow \quad -\frac{1}{2}\cos 2p + \frac{1}{2} = \frac{1}{4}$$

$$-\frac{1}{2}\cos 2p = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$$

$$\cos 2p = \frac{2}{4}$$

$$\cos 2p = \frac{1}{2}$$



$$2p = \frac{\pi}{3},\ 2\pi - \frac{\pi}{3}$$

$$2p = \frac{\pi}{3},\ \overset{+2\pi}{\frac{5\pi}{3},\ \frac{7\pi}{3}}$$

$p = \frac{\pi}{6}, \frac{5\pi}{6}, \cancel{\frac{7\pi}{6}}$ out of range $0 \le p < \pi$

12B

8 (a) $\cos 2x = 1 - 2\sin^2 x$

$$\cos 2x + 2\sin^2 x = 1$$

$$2\sin^2 x = 1 - \cos 2x$$

$$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$$

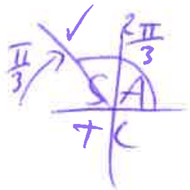
(b) $\displaystyle\int_{\pi/4}^{\pi/3} \sin^2 x \, dx$

$$= \int_{\pi/4}^{\pi/3} \frac{1}{2} - \frac{1}{2}\cos 2x \, dx$$

$$= \left[\frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{2}\sin 2x\right]_{\pi/4}^{\pi/3}$$

$$= \left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]_{\pi/4}^{\pi/3}$$

$$= \left(\frac{1}{2}\cdot\frac{\pi}{3} - \frac{1}{4}\sin\left(2\frac{\pi}{3}\right)\right) - \left(\frac{1}{2}\cdot\frac{\pi}{4} - \frac{1}{4}\sin\left(2\frac{\pi}{4}\right)\right)$$



$$= \left(\frac{\pi}{6} - \frac{1}{4}\sin\left(\frac{\pi}{3}\right)\right) - \left(\frac{\pi}{8} - \frac{1}{4}\sin\left(\frac{\pi}{2}\right)\right)$$

$$= \left(\frac{\pi}{6} - \frac{1}{4}\cdot\frac{\sqrt{3}}{2}\right) - \left(\frac{\pi}{8} - \frac{1}{4}\right)$$

$$= \left(\frac{\pi}{6} - \frac{\sqrt{3}}{8}\right) - \left(\frac{\pi}{8} - \frac{1}{4}\right)$$

$$= \frac{1}{24}(4\pi - 3\sqrt{3}) - \frac{1}{24}(3\pi - 6)$$

$-(-6) = +6$

$$= \frac{1}{24}\left(4\pi - 3\sqrt{3} - 3\pi + 6\right) = \frac{1}{24}\left(\pi + 6 - 3\sqrt{3}\right)$$